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On Nonlinearity of Transitional Boundary-Layer Flows

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On nonlinearity of transitional boundary-layer flows

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This paper focuses on two specific effects concerning the nonlinearity of transitional boundary-layer flows: the generation of turbulent spots in a flat-plate flow, and the nonlinear behaviour of laminar separation bubbles associated with the short-scale instability of the separated layer. A compilation of results from several experimental studies is given.

1. Introduction

Transitional boundary-layer flows represent a variety of quite different nonlinear effects. The aim of this paper is to illustrate two of these effects. The first is the generation of turbulent spots in a flat-plate boundary layer, which may result from the interaction between boundary-layer disturbances of different types. The second is the nonlinearity of local boundary-layer separation regions associated with the backward influence of disturbances growing behind the separation point. These topics are discussed from an experimental viewpoint in the light of recently published data.

2. A mechanism for turbulent spot generation in a flat-plate boundary layer

(*a*) *Preamble*

The complex phenomena of boundary-layer transition is comprised of a sequence of events; starting with the excitation of laminar boundary-layer perturbations and finishing with the breakdown to turbulence. The last stage of this process, following the evolution of small-amplitude disturbances, represents a number of finite-amplitude effects. It is just this nonlinear stage of the transition that is responsible for the onset of chaos in initially organized flow and it is also the most complex.

Klebanoff *et al.* (1962) were the first to study the nonlinear breakdown in the flat-plate boundary layer in detail. The spanwise periodicity of the flow in the transitional region, the rapid growth of harmonics of the fundamental disturbance and the appearance of 'spikes' correlated to the period of the primary wave were the features of the transition examined in that work, which is referred to now as the K-type of breakdown. The other evolutionary path of the boundary-layer transition, represented by the gradual amplification of spectral components and subharmonic generation, was observed by Kachanov *et al.* (1977). The details of this type of transition were then investigated by Kachanov & Levchenko (1984). In Saric *et al.* (1984), by varying the initial amplitudes of the primary wave and its subharmonic, both of the regimes were reproduced and examined with the emphasis on the spatial arrangement of the 3D transitional vortical structures, i.e. the L-shaped vortices, in each case. Two paths of the transition, similar to those found in the flat-plate boundary layer, were also observed in plane Poiseuille flow by Kozlov & Ramazanov (1983, 1984). Thus, a number of fundamental features of the nonlinear breakdown were revealed in the above-mentioned reference along with other studies on the forced boundary-layer transition, in which the vibrating-ribbon technique for excitation of controlled disturbances were used.

A basic nonlinear transitional effect, which is not reproduced by this experimental technique, marks the onset of turbulence owing to the generation of turbulent spots, and is observed in experiments for 'natural' conditions (see, for example, Arnal & Juillen 1977). The characteristics and interactions of turbulent spots have been studied in several papers (see, for example, Emmons 1951; Wygnanski *et al.* 1976). The method used in these studies consists of experimentally modelling the turbulent spots, which were artificially produced by some sort of strong local inhomogeneity inserted into the boundary layer (Wygnanski *et al.* 1976; Barrow *et al.* 1984).

This section focuses on a mechanism for the generation of turbulent spots, which may occur naturally. One possiblity is the interaction of a localized boundary-layer disturbance with a Tollmien–Schlichting (TS) wave. This is considered in what follows.

(*b*) *Experimental data*

First, the transformation of a localized disturbance into the turbulent spot of a flat-plate boundary layer is shown in figure 1 (Grek *et al.* 1990): the smoke-wire visualization of this process was obtained from a series of photos and shows a sucessive increase of the streamwise coordinate. The disturbance is excited through the small hole of the flat-plate surface placed at $Re^* = 1.72(U_\infty x/\nu)^{1/2} = 470$, where U_∞ is the external-flow velocity. Further downstream it produces the nonlinear wavepacket, or L-shaped vortex, which then transforms into the turbulent spot. Evolution of the nonlinear wavepacket is accompanied by peripheral 3D waves visualized clearly in the photos. Their inclination to the mean-flow direction increases while propagating downstream, reaching a maximum value at the formation of the spot. The wave pattern of figure 1 probably just corresponds to the generation of the wavepacket of 3D TS waves associated with the turbulent spot that was found in other experiments (Wygnanski *et al.* 1976). It was found that increasing the initial amplitude of the localized disturbance resulted in the acceleration of the turbulent-spot generation, while decreasing the amplitude caused the perturbation to dissipate and the spot did not form.

However, the disturbance with an amplitude lower than the threshold of the above generation process can also produce the turbulent spot by interaction with the TS wave (Grek *et al.* 1991). The streamwise amplitude variation of the boundary-layer disturbances is shown for this case in figure 2. The coordinate of the localized excitation is $Re^* = 470$ and the reduced frequency of the TS wave is $F = 2\pi f\nu/U_\infty^2 = 273 \times 10^{-6}$. Both the localized perturbation and the TS wave generated by a vibrating ribbon decay if they are excited separately, but at the interaction the localized perturbation grows and, finally, the turbulent spot appears. This is illustrated in figure 3 by the phase-correlated time traces of the hot-wire signal. In the absence of the TS wave, the localized perturbation dissipates (figure 3*a*) but at the interaction it transforms into the wavepacket, with a central frequency

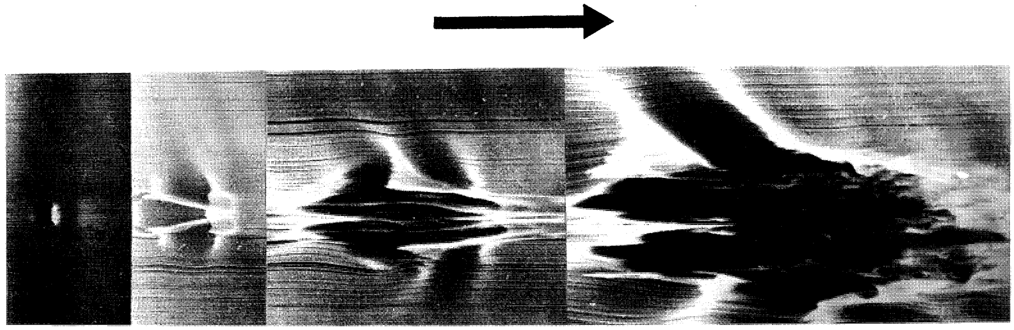

Figure 1. Generation of the turbulent spot in Blasius flow, plane view: 1, localized disturbance; 2, Λ-shaped vortex; 3, turbulent spot.

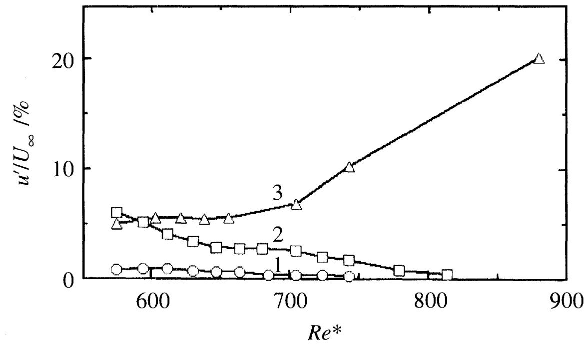

Figure 2. Amplitude variation of disturbances: 1, TS wave; 2, localized disturbance; 3, at the interaction.

close to half that of the TS wave (according to the data of spectral measurements), and then into the turbulent spot (figure 3*b*). The same result was obtained qualitatively when modelling the interaction of the localized disturbance with a spatial wave packet of 3D instability waves generated by small-amplitude harmonic excitation through the small hole of the surface. The difference with the above case is that higher initial amplitudes of the boundary-layer waves are required to produce the turbulent spot because of the spreading of the wavepacket of instability waves in the spanwise direction.

To classify the localized perturbation used in that study, it was compared to disturbances with documented characteristics which were attributed to transitional boundary layers. Its propagation velocity is equal to 0.6–0.8 times that of the external flow. The amplitude distribution, with the maximum near the edge of the boundary layer, and gradual decay in the streamwise direction were found to closely resemble the disturbance examined by Grek *et al.* (1985) and called 'puff' following the terminology of Wygnanski *et al.* (1975) for the pipe flow. Thus, the next step of the study was to reproduce a puff, as in Grek *et al.* (1985), and to examine its interaction with the TS wave in a similar way to the above experiment.

Figure 4 shows streamwise variations of the amplitudes for the puff, TS wave and the disturbance which originates from their interaction. Again, the first two decay when excited separately but when interacted with each other they produce the growing perturbation. Its generation is shown by the set of oscilloscope traces in

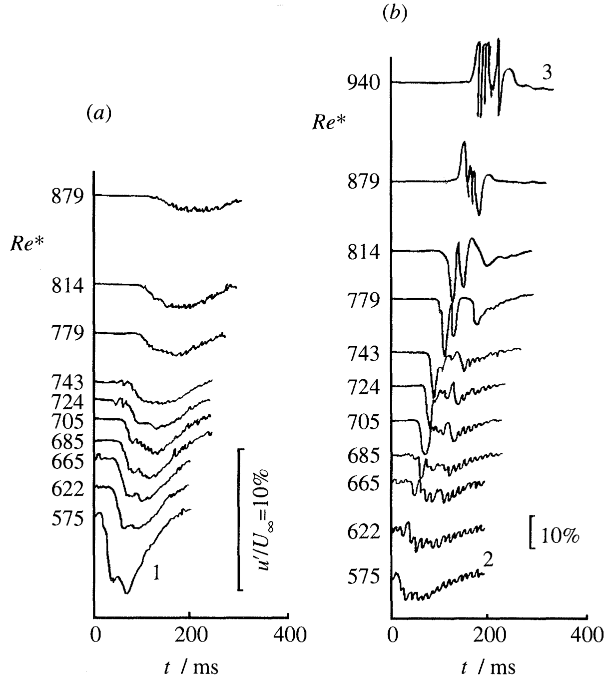

Figure 3. Time traces for: (*a*) the isolated localized disturbance; (*b*) the interaction with a 2D TS wave, $F = 273 \times 10^{-6}$. 1, localized disturbance; 2, TS wave; 3, turbulent spot.

figure 5. The puff-to-TS wave interaction gives rise to the wavepacket which forms, like that in the above experiment, around the subharmonic of the TS wave. Streamwise evolution of the wavepacket and its further transformation into the turbulent spot proceed independently of the puff (propagation velocities of the wavepacket and the puff are equal to 0.42 and 0.76, respectively) and the TS wave, which dissipates after the interaction. The position at which generation of the wavepacket starts is compared to the neutral curve for linear instability of a flat-plate boundary layer in figure 6. In summary, the above experimental data reveal one of the possible nonlinear mechanisms involved in the boundary-layer transition. It consists of the interaction of the perturbations inherent to a transitional flow, i.e. linear-instability waves and localized puff-like disturbances, producing the wavepacket which then transforms into the turbulent spot.

3. Feedback effects in transitional separating flows

(*a*) *Preamble*

This section focuses on another specific nonlinear feature of transitional boundary-layer flows, i.e. interactions of disturbances with the mean flow and background perturbations in laminar boundary-layer separation regions. A sketch of a local nominally 2D separated flow, i.e. a separation bubble, which is considered here, is shown in figure 7. Under ordinary conditions a laminar separating layer represents the instability which results in the transition occurring in the bubble, or nearby. In this case, the flow pattern in the whole bubble depends on the transition that was found in the experiments of Chapman *et al.* (1958) and pointed out in calculations of transitional separation bubbles (see, for example, Vatsa & Carter 1984). A manifestation

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Figure 5

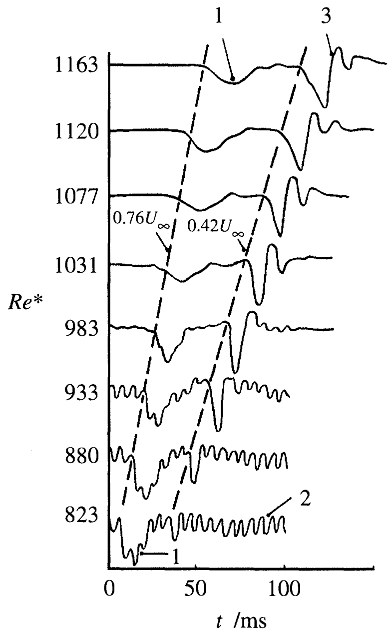

Figure 4

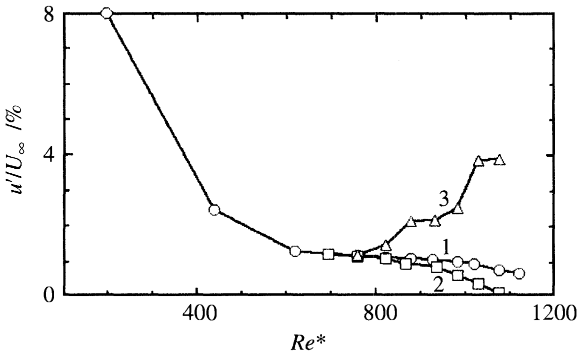

Figure 4. Amplitude variation of disturbances. 1, puff; 2, TS wave, $F = 155 \times 10^{-6}$; 3, at the interaction.

Figure 5. Interaction of the puff with the TS wave, $F = 155 \times 10^{-6}$. 1, puff; 2, TS wave; 3, wave packet.

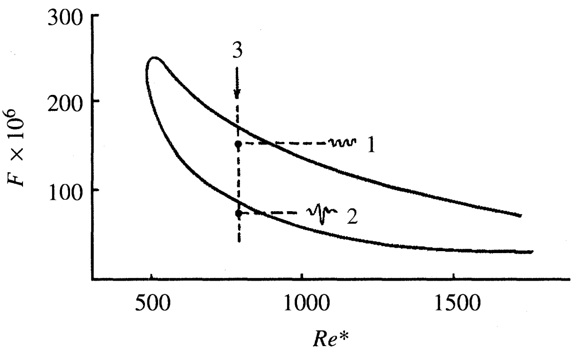

Figure 6. Puff-to-TS wave interaction and the neutral-stability curve. 1, TS wave; 2, the wave packet; 3, indicates the section in which the interaction begins.

of this upstream influence of the transition on the separation region is the variation of the mean-flow pattern in the laminar part of the separation bubble, caused by excitation of the linear shear-layer instability (figure 8). Small-amplitude growing waves generated in the separated region stimulate the transition and, thus, produce the disturbance of the natural laminar-flow profile ΔU, which can be an order of magnitude larger than the local amplitude of the oscillations u'_{f} (Dovgal *et al.* 1987).

To a first approximation, this nonlinear behaviour of laminar separated flows can be reduced to a correlation of the flow pattern in the separation bubble to the location of the transition point that is implied in the calculations of separation bubbles with prescribed transition (Crimi & Reeves 1976; Roberts 1980; Vatsa & Carter

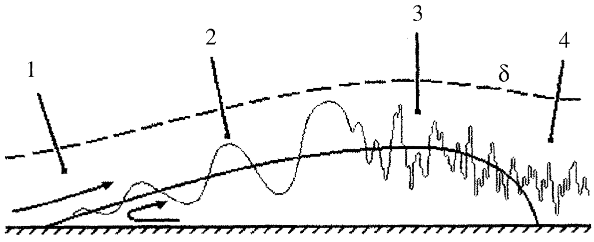

Figure 7. Sketch of a separation bubble. S, laminar separation; R, turbulent reattachment. 1, laminar boundary layer; 2, instability of the separated layer; 3, location of the transition to turbulence; 4, turbulent flow.

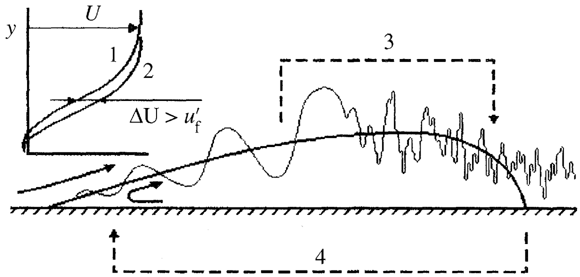

Figure 8. Backward influence of disturbances in a separation bubble. 1 and 2, natural mean-flow profile and that under excitation, respectively; 3, dashed line indicates the disturbance of the reattaching flow produced by growing waves; 4 dashed line indicates the upstream influence of the reattaching flow upon the laminar part of the separation bubble.

1984; Davis *et al.* 1987). Actually, in this way, the priority of the transition 'point' for characteristics of the separation bubble is assumed. However, the 'point' of the transition is rather an engineering simplification, while what is of real importance for upstream interactions is the nonlinearity of disturbances in the extended transitional region. The details of their effect upon the separated flow are described in Boiko *et al.* (1990, 1991).

(*b*) *Effect of the disturbances on the mean flow*

In the study of Boiko *et al.* (1990), the effect of the transition on the mean flow at a 2D hump immersed in the flat-plate boundary layer was examined. Under experimental conditions, the separation bubble downstream of the hump and the natural transition were separated from each other, so that laminar separation was followed by laminar reattachment, while the transition took place further downstream in the attached boundary layer. By using a 2D turbulator, placed in the boundary layer downstream of the separation bubble, to force the transition, the minimum streamwise coordinate of the transition point was found, for which the separated flow was unaffected by turbulization of the boundary layer. The streamwise growth of disturbances in the flow, indicating the positions of the transition in both cases—which is identified, in the usual way, by the location of the maximum amplitude of the perturbations u'—is shown in figure 9. The data are given for a rectangular hump of height h, related to the local boundary-layer thickness by $h/\delta = 0.35$, placed at $Re_x = U_\infty x/\nu = 1.76 \times 10^5$ from the flat-plate leading edge and at $Re_h = U_\infty h/\nu = 730$. Thus, movement of the transition point within these limits (curves 1 and 2 in the figure) does not have any influence upon the mean flow

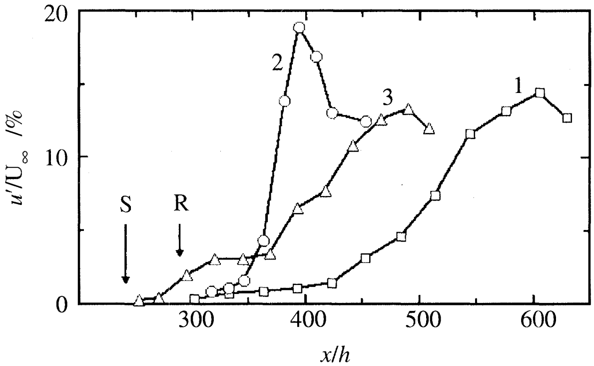

Figure 9. Streamwise growth of disturbances behind the 2D hump of a flat-plate surface. 1, natural transition; 2, forced transition; 3, transition at the excitation of the separated-flow instability. Separation point is at $x/h = 242$. The position of the reattachment is at $x/h = 285$–290.

of the separation bubble. Otherwise, when the transition is stimulated by artificial excitation of the instability in the region of separation, the mean-flow variation ΔU in the separation bubble takes place, see figure 8, although the transition point is in some intermediate position—curve 3 in the figure. This check indicates that the flow pattern in the separation bubble is not correlated to the transition point but depends on the upstream evolution of the disturbances. To experimental accuracy, the effect of the instability waves on the separated flow is found when their amplitude at the reattachment, after preliminary amplification in the separation region, is not smaller than about 1%.

Further fine backward influence of the perturbations on the mean flow has been observed by Boiko *et al.* (1991) for boundary-layer separation at a small step of the flat-plate surface. In that work, a totally laminar flow was reproduced under natural conditions, i.e. downstream of the reattachment, the Blasius boundary layer was recovered without further transition to turbulence. Even in this case, the disturbances excited in the separation bubble of high enough amplitude caused a similar effect. In figure 10, the streamwise growth of 2D instability waves, artificially generated with different initial amplitudes, is shown. The data are given for the backward-facing step at $h/\delta = 0.39$, $Re_x = 2.105$ and $Re_h = 880$. The disturbances amplify in the separation region and then decay in the attached boundary layer. Full symbols indicate the amplitudes of the disturbances which do not affect the mean flow and open symbols denote those amplitudes at which the mean-flow rearrangement of the whole bubble is observed. Here, the threshold amplitude for the upstream interaction near the position of the reattachment is also about 1%. This value seems to be the lower amplitude limit for which a nonlinear wave produces a local disturbance of the mean flow which spreads all over the separation region. The correlation between oscillation amplitudes at the reattachment and the U-variation—which is maximum at the inflection point of the mean-flow profile—measured in the bubble close to the separation is shown in figure 11. Apparently, in the laminar flow, the backward influence of disturbances excited in the separation bubble is smaller than in the previous case and, moreover, in transitional separation regions. In conclusion, even though the instability does not initiate the transition, the mean-flow pattern in the laminar separation region may not be identical to that which is considered as being stationary.

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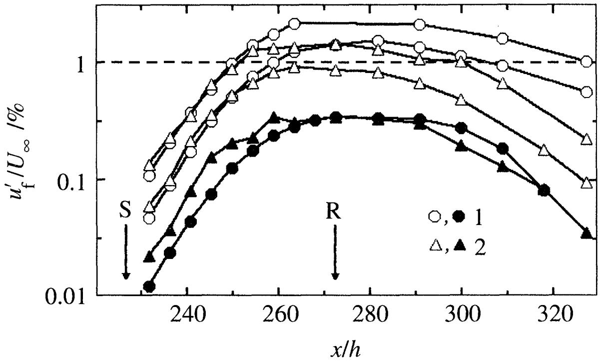

Figure 10. Streamwise growth of disturbances behind the backward-facing step of a flat-plate surface. 1, oscillations frequency $F = 207 \times 10^{-6}$; 2, $F = 249 \times 10^{-6}$. Separation point is at $x/h = 227$. The position of the reattachment is at $x/h = 270$–275.

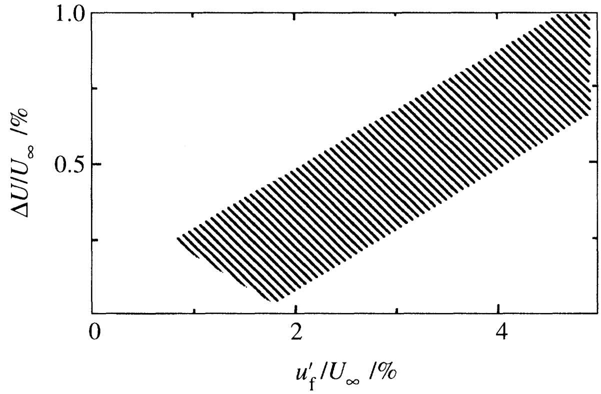

Figure 11. Correlation between amplitudes of instability waves at the reattachment, $x/h = 270$, and the mean-flow variation measured close to the separation point at $x/h = 232$. The dashed area covers the data for the frequencies of disturbances between $F = 119.10 \times 10^{-6}$ and $F = 249 \times 10^{-6}$.

(*c*) *Effect of the disturbances on the background unsteadiness*

The other feedback effect of nonlinear disturbances consists of the suppression of background noise under artificial excitation of instability waves in a separation region. This is illustrated in figure 12 for a separation bubble behind a 2D hump of a flat-plate surface (Boiko *et al.* 1991). Experimental parameters for this configuration are $h/\delta = 0.39$, $Re_x = 1.52 \times 10^5$ and $Re_h = 760$. In figure 12, two fluctuation spectra, measured at the same point close to the separation, are shown—one of the spectra is for natural conditions and the other is under the excitation of the instability wave. Generation of the disturbance is followed by a decrease in the low-frequency background fluctuations. The suppression of the irregular spectrum is observed just behind the separation point, where in all other respects the behaviour of the excited wave is that of the infinitesimal disturbance.

This excitation effect on the natural spectrum should be attributed, again, to the backward influence in the separation bubble. In figure 13, low-frequency fragments of the spectra are shown for two cases: the excitation of (*a*) the harmonic disturbance by a vibrating ribbon in the boundary layer upstream of separation and (*b*) in the separation bubble. The oscillations introduced by a vibrating ribbon into the separation region behind the point of observation, being a streamwise-propagating

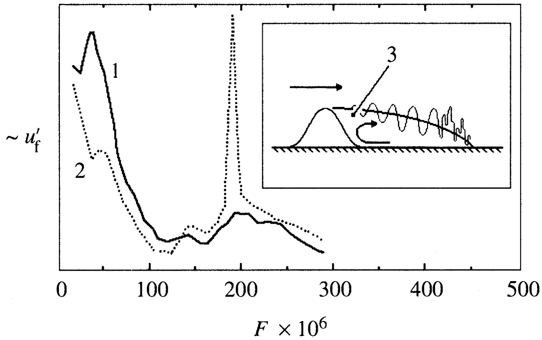

Figure 12. Suppression of the background fluctuations in the separation bubble. 1, natural spectrum; 2, excited spectrum, peak value of u'_f is 0.22% to U_∞; 3, the observation point.

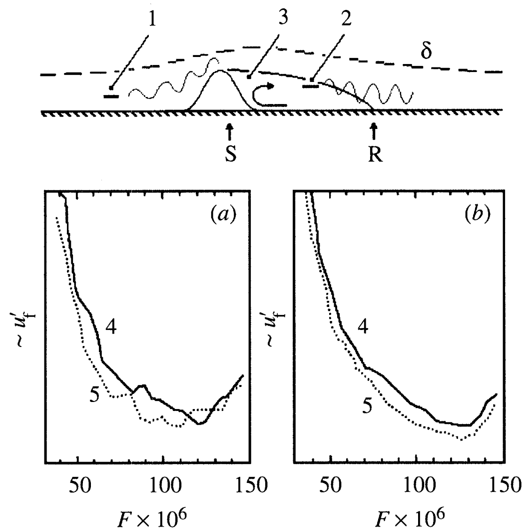

Figure 13. Low-frequency part of the spectra. (*a*) excitation by the vibrating ribbon upstream of the separation, 1; (*b*) excitation by the vibrating ribbon in the bubble, 2; excitation frequency $F = 285 \times 10^{-6}$; 3 indicates the observation point; 4, natural spectra; 5, under the excitation.

wave, have the same influence upon the spectrum as those generated in the boundary layer before the separation. In all probability, these observations correspond to the fact that the 'initial' spectrum of fluctuations near to the separation includes the response of the separation bubble to environmental perturbations as well as to high-amplitude disturbances of the reattaching flow, while the latter are affected by the excitation.

To conclude, we point out that theoretical and experimental data on the transition in separation bubbles indicate that—similar to the transition in other boundary-layer flows—it starts from the streamwise growth of small-amplitude disturbances, and the initial stage of the transition can be well predicted by linear-stability analysis (Dovgal *et al.* 1994). At the same time, amplifying linear-instability waves represents specific nonlinear features shown by the above data that is peculiar to local boundary-layer separation regions.

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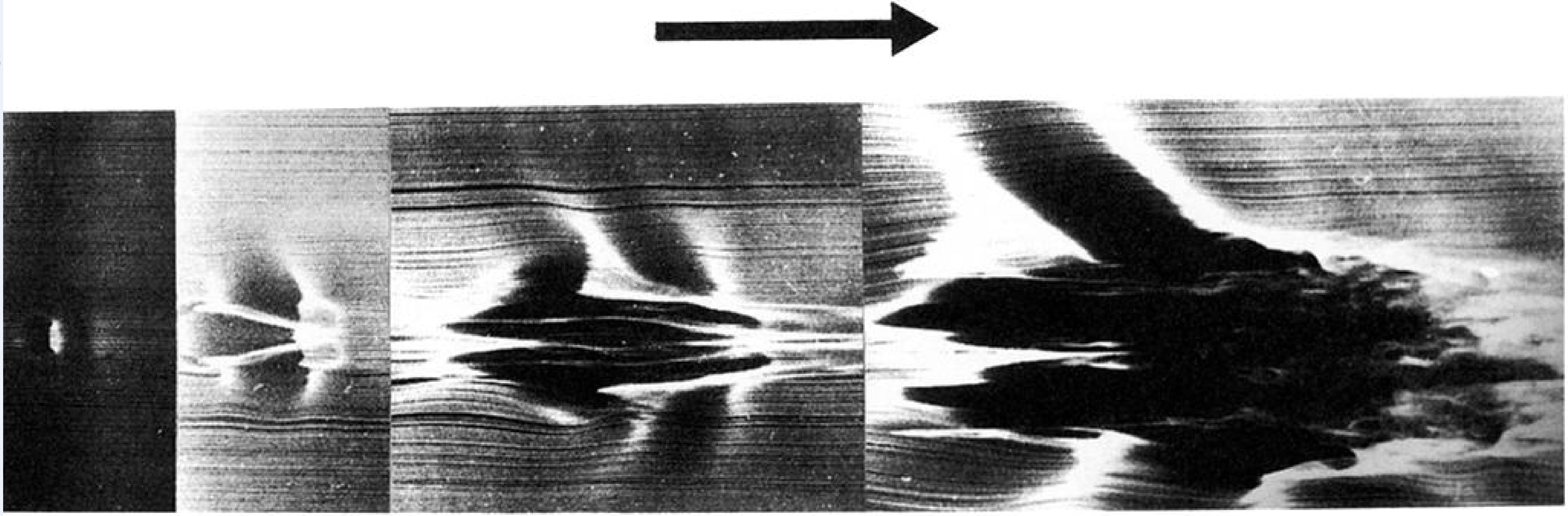

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